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GACR

+3, 1st SEMESTER EXAMINATION-2018
(SCIENCE)

Sub: MTC (Math) Full Marks: 60

Paper: CORE-I Time: 3 Hours

Answer the questions as per instruction.

The figure in the right hand margin indicate marks.

Answer any SIX questions from Q. No.1 and any FOUR from the rest.

1. (a) Find the asymptotes to the curve $y = \tan x$ [2x6]

(b) Solve $\lim_{x \to 0} \frac{1-\cos 3x}{x^2}$ using L-hospital rule.

(c) Find the radius of curvature of a straight line.

(d) Find the point of inflexions for the curve

$f(x,y) = x^2 - 6xy + 2y^2$

(e) State the reduction formula of $\int \sin nx.dy$.

(f) State the general equation of cylinder.

(g) Define right circular cone.

(h) Define scalar triple product of three vectors.

2. (a) Find the maximum and minimum values of [6]

$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

(b) If V = cos 3x cos 4y sin n 5z, then prove that [6]

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$$

3. (a) Find the asymptotes parallel to co-ordinate axes for the curve $x^2y^2 - a^2y^2 - b^2x^2 + 2xy - 3y + 1 = 0$ [6]

   (b) Find the radius of curvature to the curve $y^2 = 2x(3-x^2)$ [6]

4. (a) If $y = x^2 \sin x$, then find the nth derivative of y using Leibritz rule. [6]

   (b) Trace the curve $y = (x+2)(x-3)$ [6]

5. (a) Evaluate $\int_0^{\pi/3} \cos^5 x.dx$ using reduction formula. [6]

   (b) Find the arc length of the curve $r = a(1+\cos\theta)$, $0 < \theta < \pi/3$ [6]

6. (a) Find the surface area in the first octant cut from the cylindrical surface $x^2 + y^2 = a^2$ by the plane z=x. [6]

   (b) Evaluate $\lim_{x \to y} \dfrac{\cos^2 x - \cos^2 y}{x^2 - y^2}$ [6]

7. (a) Find the equation of cone whose vertex is at origin and guiding curve is $\dfrac{x^2}{9} + \dfrac{y^2}{4} + z^2 = 1$, $x + 2y + 3z = 6$. [6]

   (b) A particle moves along the curve $\vec{r} = (t^3 + 2t)\hat{i} + (t^2 + 7t)\hat{j} + (4t^2 + 2t^3)\hat{k}$. Where t denotes the time. Find the magnitude of acceleration along the tangent and normal. [6]

8. (a) Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$ [6]

   (b) Evaluate the line integral $\int_c (y^2dx - 2x^2dy)$ [6]

   where c: the parabola $y = x^2$ from (0, 0) to (2, 4)

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(SCIENCE)

Sub: PHYSICS Full Marks: 60

Paper: CORE-I Time: 3 Hours

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1. (a) Find the integrating factor (I.F) of $L\dfrac{di}{dt}+RI=E$. [2x10]

(b) $\int_{-\infty}^{\infty}\delta(x-a)\delta(x-b)dx=?$ Filll the blank.

(c) A vector under rotation is given by $\vec{r}\,'=R(\theta)\vec{r}$.

(d) $\vec{F}$ is a vector field and it is a conservative force field. Find $\vec{\nabla}\times\vec{F}$

(e) If $\vec{r}$ is a position vector in 3D, then what is the value of grad $\dfrac{1}{r}$.

(f) What is a base vector in curvilinear co-ordinates?

(g) Find the surface integral $\iint_{s}\vec{r}.d\vec{s}$ if surface $\vec{s}$ encloses volume V^{s}

(h) State stoke's theorem in plane.

(i) Integrate $\int_{0}^{1}\int_{0}^{1}\vec{F}.d\vec{l}$ along the path $y=x^2$ for $\vec{F}=x\hat{i}+y\hat{j}$

(j) If $u=x^2y$, where $x^2+xy+y^2=1$, Find $\dfrac{du}{dx}=?$

(P.T.O...)

2. (a) Define Dirac delta function. [2]

(b) Show that $\delta(a^2 - x^2) = \frac{1}{2|a|}[\delta(a-x) + \delta(a+x)]$. [3]

(c) Starting from $\delta(x) = \int_{-\infty}^{\infty} e^{ikx} dk$, show that [3]

$$\delta(x) = \lim_{\sigma \to \infty} \left(\frac{\sin \sigma x}{\pi x} \right).$$

OR

(a) How can you make an inexact diferential it exact diferential? [2]

(b) Integrate the differential

$dz = (8e^{4x} + 2xy^2)dx + (4 \cos 4y + 2x^2y)$ dy using the exact of differentials. [6]

3. (a) What is a scale factor in curvilinear co-ordinates. [2]

(b) Derive the expression for $\nabla^2\phi$ in spherical polar co-ordinates. [6]

OR

Derive the expression for velocity and acceleration in spherical polar-co-ordinates. [3+5]

4. (a) Discuss the co-ordinate transformation under rotation in 3D and show that $x_i^1 = \sum_{j=1}^{3} a_{ij} x_j$ and $\sum_{k=1}^{3} \delta_{ik}\delta_{jk}$ for $i, j = 1, 2, 3$. [4]

(b) Show that the dot product of two vectors under rotation is invariant [4]

OR

(a) Show that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$. [3]

(b) $[\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b} = [\vec{a}\,\vec{b}\,\vec{c}]^2$ [3]

(c) Explain geometrical meaning of $\vec{a} \times (\vec{b} \times \vec{c})$ [2]

5. (a) Prove that $\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.\vec{\nabla} \times \vec{A} - \vec{A}.\vec{\nabla} \times \vec{B}$ [4]

(b) Prove that $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A}$ [4]

OR

Derive the expression for curl in spherical and cylindrical polar co-ordinates. [4+4]

6. (a) State and explain stokes theorem in 3D and reduce it in X-Y- plane (Green's theorem) [3+2]

(b) Verify stoke's theorem in plane for $\oint_c (xy + y^2)dx + x^2 dy$, where c is a closed curve of region bounded by $y=x^2$ and $y=x$. [3]

OR

Evaluate $\iiint_v (\nabla.F)dv$, where v = volume enclosed region bounded by the planes $x=0, y=0, z=0$ and $2x + 2y + z = 4$ for vector $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ [8]

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GACR

+3, 1st SEMESTER EXAMINATION-2018 (SCIENCE)

Sub: MATHEMATICS Full Marks: 60

Paper: CORE-I Time: 3 Hours

Answer the questions as per instruction.

The figure in the right hand margin indicate marks.

Answer any SIX questions from Q. No.1 and any FOUR from the rest.

1. (a) Find the asymptotes to the curve $y = \operatorname{cosec} x$ [2x6]

(b) Evaluate $\lim_{x \to 0} \dfrac{2\tan x + x}{3x + 2x^2}$

(c) Write down the formula for radius of curvature in polar form.

(d) If $y = x.\sin x$, then find $\dfrac{d^n y}{dx^n} = ?$.

(e) if $\vec{A} = t^2\hat{i} - t\hat{j}$, $\vec{B} = 2t\hat{j} + 5t^2\hat{k}$,

$\vec{C} = 3t\hat{i} + 2t^2\hat{k}$ then find $\dfrac{d}{dt}\left[\vec{A}\vec{B}\vec{C}\right]$

(f) Evaluate $\int_0^{\pi/3} \sin^3 t.\cos t\, dt$

(g) Write down polar equation for sphere.

(h) Find the arc length of the curve $\vec{\gamma} = t^2\hat{i} - 3t\hat{j} + t^3\hat{k}$ from (1,-3,1) to (4, -6, 8).

(P.T.O...)

2. (a) Use L-Hospital Rule to find $\lim_{x \to 0} \frac{x^3 - 2x^2}{\sin^2 x}$ [6+6]

(b) If $y = e^{ax} \cos x$ then find $\frac{d^n y}{dx^n}$

3. (a) Find the radius of curvature for the curve $y^2 = 2x(3-x^2)$ at x=1 [6+6]

(b) Trace thee curve $r = a(1 + \cos\theta)$

4. (a) Find the asymptotes parallel to co-ordinate axes for the curve $x^2y^2 - a^2x^2 - 16y^2 + 2xy - 3x^2 = 0$ [6+6]

(b) Derive the reduction formula for $\int \sec^n x.dx$

5. (a) Find the equation of the cone whose vertex is at (2,1,3) and guiding curve is $x^2 + y^2 = 16, z = 0$ [6+6]

(b) Find the circle of curvature to the parabola $y^2 = 4ax$ at (a, a)

6. (a) Find the equation of cylinder having guiding curve $4x^2 - 9y^2 = 36$, z = 2, and whose generator are parallel to the line $\frac{x-1}{z} = \frac{y+2}{3} = \frac{z-1}{1}$. [6+6]

(b) Find the scalar and vector triple products of the vector $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = 3\hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$

7. (a) If $\vec{r}(t) = e^{2t}\sin t\,\hat{i} + e^{2t}\cos t\,\hat{j} + e^{2t}\hat{k}$, then find $[\vec{r}, \vec{r}', \vec{r}'']$. (Scalar triple product). [6+6]

(b) Find the volume of the paralleo piped whose three adjacent sides are $\vec{a} = 3\hat{i} - 4\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$, and $\vec{c} = 2\hat{i} + 2\hat{j} + 3\hat{k}$.

8. (a) Use reduction formula to evaluate $\int \cos'' x.dx$. [6+6]

(b) Find the radious of curvalate for the following curve $x = 6t^2 - 3t^4,\ y = 8t^3$

- x - x - x -