REAL ANALYSIS

CORE PAPER - III

1 mark questions

1. Define the algebraic properties of R.
2. What is the ε-neighborhood of a point in R?
3. Define a set that is "bounded above."
4. Define a set that is "bounded below."
5. What are bounded and unbounded sets?
6. Define supremum and infimum of a set.
7. Explain the completeness property of R.
8. What is the Archimedean property?
9. Explain the density of rational numbers in R.
10. Explain the density of irrational numbers in R.
11. Define the interval of a set.
12. What is an interior point?
13. Define open and closed sets.
14. What is a limit point of a set?
15. Explain the Bolzano-Weierstrass theorem for sets.
16. Define the closure of a set.
17. Define the interior of a set.
18. Define the boundary of a set.
19. Define sequences and subsequences.
20. What is a bounded sequence?
21. What is a convergent sequence?
22. Define the limit of a sequence.
23. State one of the limit theorems.
24. What is a monotone sequence?
25. Define divergence criteria.
26. Explain the Bolzano-Weierstrass theorem for sequences.
27. What is a Cauchy sequence?
28. Explain Cauchy's Convergence Criterion.
29. What is an infinite series?
30. Define convergence and divergence of an infinite series.
31. Explain the comparison test for convergence.
32. Explain the limit comparison test.
33. Define the ratio test for convergence.
34. What is Cauchy's nth root test?
35. Explain the integral test for convergence.
36. Define an alternating series.
37. What is the Leibniz test for convergence?
38. Define absolute and conditional convergence.
39. Explain the epsilon-delta approach to limits of functions.
40. What is the sequential criterion for limits?
41. State a limit theorem.
42. Define one-sided limits.

43. What are infinite limits and limits at infinity?
44. Define continuous functions.
45. What is the sequential criterion for continuity and discontinuity?
46. What is the Boundedness Theorem for continuous functions?
47. Define Bolzano's Intermediate Value Theorem.
48. What is the theorem about the location of roots?
49. Define uniform continuity.
50. What is the Mean Value Theorem?
51. The __________ property of R states that every non-empty subset of R that is bounded above has a least upper bound.
52. An __________ neighborhood of a point in R is an open interval centered at the point.
53. A set that is bounded above has an upper bound, denoted as __________.
54. A set that is bounded below has a lower bound, denoted as __________.
55. If a set is bounded, it has both upper and lower bounds, otherwise it is __________.
56. The supremum of a set is the smallest upper __________.
57. The infimum of a set is the largest lower __________.
58. The __________ property states that for any positive real number, there is a natural number larger than it.
59. The __________ of rational numbers in R refers to the fact that between any two real numbers, there is a rational number.
60. The __________ of irrational numbers in R refers to the fact that between any two real numbers, there is an irrational number.
61. An interval is a set of numbers with the property that any number that lies between two numbers in the set is also considered a(n) __________.
62. An __________ point of a set is a point all of whose neighborhoods contain points from the set.
63. __________ sets in R are those that contain their boundary, while __________ sets are those that do not.
64. A __________ point of a set in R is a point that can be "approached" by points of the set.
65. The __________ theorem states that every bounded sequence in R has a convergent subsequence.
66. A __________ is a list of numbers, while a __________ is a subset of a sequence.
67. A __________ sequence is one that is bound above and below.
68. A __________ sequence is one that tends to a limit.
69. The __________ of a sequence is the number that the sequence approaches as n approaches infinity.
70. __________ sequences are those that either increase or decrease but do not do both.
71. If a sequence does not approach a finite limit, it is __________.
72. The __________ theorem for sequences is a key result in real analysis stating that every bounded sequence has a convergent subsequence.
73. A __________ sequence is a sequence such that the difference between any two terms gets arbitrarily small as the sequence progresses.
74. The __________ convergence criterion is a condition that can be used to prove a sequence is convergent.
75. An __________ series is a sum of infinitely many terms.

76. The __________ test compares the terms of a series to those of a known series.
77. The __________ test compares the ratio of successive terms of a series to a known limit.
78. The __________ test checks the ratio of the nth root of the nth term of a series to a known limit.
79. The __________ test checks if the terms of a series decrease at least as rapidly as those of a known series.
80. An __________ series alternates between positive and negative terms.
81. The __________ test checks whether the absolute value of terms in a series decrease.
82. The __________ approach to limits of functions gives a formal definition of limits.
83. The __________ criterion for limits uses sequences to define limits of functions.
84. __________ limits approach a value from one direction only.
85. __________ limits are those where the value of the function becomes arbitrarily large as the input approaches a certain value.
86. __________ functions do not have any breaks, jumps, or points where the function is not defined.
87. The __________ Theorem states that a continuous function on a closed interval achieves its maximum and minimum value.
88. Bolzano's __________ Value Theorem is a result that guarantees the existence of roots for continuous functions.
89. The theorem concerning the __________ of roots states that a continuous function takes all intermediate values between its maximum and minimum.
90. A function is __________ continuous if for all $\varepsilon > 0$ there exists $\delta > 0$ such that for all x and y in the domain, if $|x - y| < \delta$, then $|f(x) - f(y)| < \varepsilon$.
91. A function is differentiable at a point if it has a __________ at that point.
92. __________'s theorem gives a condition for differentiability in terms of the limit of a certain function.
93. The __________ rule is a formula to compute the derivative of a composite function.
94. The __________ Value Theorem states that if a function is continuous over the interval [a, b] and differentiable over the open interval (a, b), then there exists some c in the interval (a, b) such that the derivative at c is equal to the average rate of change over the interval [a, b].
95. __________'s theorem is a special case of the Mean Value Theorem, applying when the function attains equal values at the endpoints of an interval.
96. The __________ theorem states that if a function is differentiable then its derivative has the intermediate value property.
97. The __________ of a function is a property that allows the function to be recovered from its derivative.
98. The __________ theorem gives conditions under which an inequality involving a function and its derivative can be proven.
99. The __________ theorem states that if a function is differentiable on an open interval, then its derivative is continuous wherever it exists.
100. __________ continuity of a function states that the function continues to approach its limit even after the limit point is passed.

2/3 marks questions

1. Let S be a set in R. If ∃M ∈ R such that ∀s ∈ S, M > s, what term is used to describe M?
2. Given a set A in R, what does it mean for A to be bounded? Provide a mathematical definition.
3. If A is a non-empty subset of R bounded above, denote the supremum of A by sup A. What does it mean for sup A to exist?
4. Given that ∀x ∈ R, ∃n ∈ N such that n > x, what property does R possess?
5. Let a, b ∈ R and a < b. Define an open interval and a closed interval in R using the given numbers a and b.
6. Given a sequence (a_n), define what it means for the sequence to be convergent to a limit L ∈ R.
7. For a sequence (a_n), define what it means for the sequence to be bounded.
8. Given a sequence (a_n) and a subsequence (a_{nk}), define what it means for (a_{nk}) to be a subsequence of (a_n).
9. For a sequence (an), define what it means for the sequence to be Cauchy.
10. Let $\sum a_n$ be an infinite series. Provide the mathematical definition of convergence for the series.
11. Define the ε-δ definition of a limit for a function f: R → R at a point c ∈ R.
12. For a function f: R → R, define what it means for f to be continuous at a point c ∈ R.
13. Given a function f: R → R, define what it means for f to be uniformly continuous over an interval I ∈ R.
14. Let f: R → R be a continuous function over a closed interval [a, b]. What does the Boundedness Theorem state about f?
15. Let f: R → R be a continuous function over the interval [a, b]. What does the Intermediate Value Theorem state about f?
16. For a function f: R → R, define what it means for f to be differentiable at a point c ∈ R.
17. Let f: R → R and g: R → R be two differentiable functions. State the Chain Rule for differentiation of the composition function (f o g).
18. State the Mean Value Theorem for a function f: R → R which is continuous over [a, b] and differentiable over (a, b).
19. Let f: R → R be a differentiable function over an interval (a, b). State Rolle's Theorem.
20. If f: R → R is a function such that f'(x) exists ∀x ∈ R, what does Darboux's theorem state about f'?

6/7 marks questions

1. Explain the algebraic and order properties of real numbers and how they lay the foundation for further mathematical analysis.
2. Discuss the concept of ε-neighborhood in R. How is this concept crucial in defining limits and continuity?

3. Differentiate between sets that are bounded above, bounded below, bounded, and unbounded. Provide examples for each.
4. Discuss the concepts of supremum and infimum. How are these concepts related to bounded sets?
5. Discuss the Archimedean Property and the Completeness Property of R. How do these properties ensure the existence of certain elements within a set?
6. What do we mean by the density of rational and irrational numbers in R? Give examples illustrating this density.
7. Differentiate between open sets and closed sets. Provide examples and discuss their significance in real analysis.
8. Define a sequence and a subsequence. How does the concept of subsequences help in understanding the properties of sequences?
9. Discuss the concept of limit of a sequence. How does the limit relate to the boundedness or convergence of a sequence?
10. Elaborate on the divergence criteria and Bolzano Weierstrass Theorem for sequences. How are these concepts related?
11. What is a Cauchy sequence? How does Cauchy's Convergence Criterion relate to the limit of a sequence?
12. Discuss the convergence and divergence of infinite series. Explain with examples.
13. Explain the different tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's nth root test, Integral test, and Alternating series test.
14. Explain the epsilon-delta approach to the limits of functions. How is this definition more rigorous and complete than the notion of limits we develop in introductory calculus?
15. Discuss the continuity and discontinuity of functions using the sequential criterion. Provide examples for better understanding.
16. Explain the Boundedness Theorem, Maximum Minimum Theorem, and Bolzano's Intermediate Value Theorem. How do these theorems utilize the concept of continuity?
17. Discuss the concept of uniform continuity and how it differs from the general concept of continuity. Provide examples.
18. Define the concept of differentiability of a function at a point and over an interval. Explain the significance of this concept in the study of calculus.
19. Discuss the Mean Value Theorem and Rolle's Theorem. How are these theorems used in the study of differential calculus and real analysis?
20. Explain the Chain Rule for differentiation. Why is this rule important when dealing with composite functions?