DEPARTMENT OF STATISTICS

CORE – XI

STOCHASTIC PROCESS

Short Type Questions

1. What is a probability generating function, and how is it used in probability distributions?
2. Explain the concept of a bivariate probability generating function.
3. Define a stochastic process and provide an example.
4. What is a stationary process in the context of stochastic processes?
5. What are the key characteristics of a stationary process?
6. How do you distinguish between a discrete and continuous stochastic process?
7. Differentiate between discrete and continuous random variables.
8. Explain the importance of the Markov property in Markov chains.
9. What is the transition probability matrix of a Markov chain?
10. How can Markov chains be represented using graphs?
11. Define a Markov chain and its key property.
12. How is the order of a Markov chain determined?
13. Discuss the concept of higher transition probabilities in Markov chains.
14. What is the relationship between Markov chains and directed graphs?
15. Explain the generalization of independent Bernoulli trials in the context of Markov chains.
16. What is meant by the classification of states in a Markov chain?
17. Differentiate between transient and absorbing states in a Markov chain.
18. How can you identify ergodic states in a Markov chain?
19. Define a periodic Markov chain and provide an example.
20. Discuss the concept of time reversibility in Markov chains.
21. List the postulates of a Poisson process.
22. What are the properties of a Poisson process?
23. Explain the concept of inter-arrival time in a Poisson process.
24. What is a pure birth process in the context of stochastic processes?
25. Define the Yule-Furry process and its characteristics.
26. Differentiate between birth and death processes.
27. What is a pure death process, and how does it differ from a pure birth process?
28. Discuss the concept of the Poisson arrival rate in Poisson processes.
29. Provide an example of a real-world application of a Poisson process.
30. How can you calculate the probability of a specific number of events occurring in a Poisson process?
31. Explain the general concept of queuing systems.

32. What are the characteristics of queuing models?
33. Define steady-state distribution in the context of queuing models.
34. What is the M/M/1 queuing model, and what does each parameter represent?
35. Discuss the differences between finite and infinite system capacity in queuing models.
36. What is the waiting time distribution in a queuing system, and why is it important?
37. Provide a brief overview of Little's law and its significance in queuing theory.
38. How does the utilization factor impact queuing system performance?
39. What are the limitations of the M/M/1 queuing model?
40. Give an example of a practical application where queuing theory is used to optimize processes.

Long Type Questions:

1. Explain the concept of generating functions in probability theory. How can generating functions be used to find moments of a random variable? Provide an example.
2. The t.p.m. of a Markov Chain $\{X_n , n = 1, 2 , \ldots .\}$ having three states 1, 2 and 3 is
$$\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$
And initial distribution is $\Pi_0 = (0.7, 0.2, 0.3)$
Find (i) $\Pr\{X_2 = 3\}$
(ii) $\Pr\{X_3 = 3, X_2 = 2, X_1 = 3, X_0 = 1\}$
3. Describe the procedure for determining higher-order transition probabilities in a Markov chain. Use a specific example to calculate and interpret these probabilities.
4. The t.p.m. of a Markov Chain $\{X_n, n = 1,2, \ldots .\}$ having states 1,2, 3 and 4 is
$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 0 & 3/4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2/3 & 1/3 \end{bmatrix}$$
Then find which states are transient or persistent.
5.
6. Define and elaborate on the bivariate probability generating function. How does it help in modeling joint distributions of two random variables?
7. Describe the fundamental characteristics of a stochastic process. Provide examples of different types of stochastic processes and explain their applications in real-life situations.
8. Define a Markov chain and discuss its key properties. Explain how the Markov property simplifies the modeling of certain random processes. Provide an example of a real-world system that can be modeled as a Markov chain.

9. Markov chains can be visualized as directed graphs. Explain how to construct a directed graph representation of a Markov chain. Illustrate this with a specific Markov chain example.
10. Discuss the concept of higher-order transition probabilities in Markov chains. How are they computed, and what information do they provide about the chain's behavior? Provide an example to demonstrate their use.
11. State the classification criteria for states in a Markov chain. Elaborate on transient states, absorbing states, and recurrent states. How does the classification impact the long-term behavior of the chain?
12. Detail the postulates of a Poisson process and explain why they are essential for modeling random events. Provide an example of a real-world process that can be approximated using a Poisson process.
13. Discuss the key properties of a Poisson process, including its independence and stationary increment properties. How are these properties useful in modeling arrival processes, such as those seen in queuing systems or customer arrivals?
14. Explain the concept of inter-arrival time in a Poisson process. How can you compute the probability density function of inter-arrival times? Provide formulas and an example to illustrate the calculation.
15. Compare and contrast different types of birth-death processes, including pure birth, pure death, and the Yule-Furry process. What are their defining characteristics, and in what scenarios are they applicable?
16. Queuing systems play a vital role in understanding and optimizing service processes. Describe the general concepts and components of a queuing system. Use a practical example to explain these concepts.
17. Explore the characteristics of queuing models, including arrival rates, service rates, and queue discipline. Discuss how these factors affect the performance and efficiency of a queuing system. Provide examples to illustrate their impact.
18. Define and explain the concept of steady-state distribution in queuing models. How can you calculate the steady-state probabilities of different states in a queuing system? Provide formulas and examples.
19. Investigate the M/M/1 queuing model in depth. Discuss the significance of each parameter (arrival rate, service rate, number of servers) and how variations in these parameters influence the system's behavior. Compare M/M/1 models with different configurations.
20. Waiting time distribution is crucial in understanding the customer experience in queuing systems. Describe how waiting time distributions are calculated and their practical importance. Discuss the limitations of waiting time distribution models.

21. Consider a random variable X with the probability mass function (PMF) given by $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, and $P(X = 3) = 0.3$. Calculate the expected value (mean) and variance of X.

22. You are given the probability generating function G(s) = 0.2s + 0.4s^2 + 0.3s^3 for a random variable X. Determine the probability mass function (PMF) of X.
23. A stochastic process represents the daily temperature at a certain location. If the process is stationary, calculate the autocovariance between the temperatures on days 1 and 5, assuming the autocovariance function is Cov(T1, T5) = 4.
24. A company produces light bulbs, and the probability of a bulb being defective is 0.1. If you randomly select 20 bulbs, what is the probability that exactly 3 of them are defective? Use the binomial probability formula.
25. Given a continuous random variable Y with probability density function (PDF) f(y) = 2y for 0 ≤ y ≤ 1, find the cumulative distribution function (CDF) of Y and calculate P(0.2 ≤ Y ≤ 0.5).
26. Consider a two-state Markov chain with the transition matrix: | 0.8 0.2 | | 0.3 0.7 | If the chain starts in State 1, what is the probability that it will be in State 2 after 3 steps?
27. A three-state Markov chain has the following transition probabilities: P(1 -> 2) = 0.4 P(2 -> 3) = 0.6 P(3 -> 1) = 0.2 If the chain starts in State 1, find the probability of being in each state after 2 steps.
28. A Markov chain has 4 states and the following transition matrix: |

$$\begin{pmatrix} 0.2 & 0.3 & 0.2 & 0.3 \\ 0.1 & 0.2 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.2 & 0.3 \\ 0.2 & 0.4 & 0.1 & 0.3 \end{pmatrix}$$

    Determine whether this Markov chain is irreducible and aperiodic.

29. A fair six-sided die is rolled repeatedly, and a Markov chain is defined based on the parity of the outcomes (even or odd). Compute the steady-state probabilities of being in the even and odd states.
30. You have a deck of cards with 52 cards. If you draw one card at a time without replacement, forming a Markov chain, calculate the probability that the first Ace appears on the 5th draw.
31. In a Poisson process with a rate of 4 arrivals per minute, what is the probability that there will be exactly 3 arrivals in a 30-second interval?
32. Given a Poisson process with a rate of 10 arrivals per hour, find the probability that there are no arrivals in a 15-minute time interval.
33. A call center receives phone calls according to a Poisson process with an average rate of 20 calls per hour. Calculate the probability that exactly 4 calls are received in a 10-minute period.
34. An earthquake occurs in a region on average once every 5 years, following a Poisson process. What is the probability that at least one earthquake will occur in a 2-year period?
35. In a Poisson process, if the expected number of arrivals in 1 hour is 3, what is the probability that the next arrival will occur within 10 minutes?
36. A fast-food restaurant serves customers with an average arrival rate of 30 customers per hour and an average service rate of 40 customers per hour. Calculate the average number of customers waiting in line and the average time a customer spends waiting in the queue.

37. A bank has two tellers, and customers arrive at an average rate of 10 customers per hour, following a Poisson distribution. Each teller takes an average of 5 minutes to serve a customer, following an exponential distribution. Calculate the average time a customer spends in the bank.
38. A car repair shop has a single service station, and customers arrive at an average rate of 15 per hour, following a Poisson distribution. The average service time is 20 minutes. Find the utilization factor, the average number of customers in the system, and the average time a customer spends in the system.
39. A software development team receives bug reports at an average rate of 5 per day. It takes an average of 2 days to fix and close a bug. Calculate the average number of open bugs and the average time a bug remains open.
40. A call center has 3 operators. Calls arrive at an average rate of 12 calls per hour, and each operator takes an average of 5 minutes to handle a call. Determine the probability that a caller has to wait for service, and calculate the average waiting time for a caller who has to wait.